Narrative: **Scalability, Capacity, and Learning Engagement (SCALE) for Fraction Face-Off**

U.S. Department of Education

Education Innovation and Research

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Mid-Phase

SCALE for Fraction Face-Off (*Absolute Priority 1* and *Absolute Priority 3*)

A. Significance

(1) National Significance of the Proposed Project

In this project, we address *Absolute Priority 1* by proposing to systematically replicate an intervention that meets the definition of "moderate evidence." We also address *Absolute Priority 3* by focusing on mathematics (abbreviated "math" in this proposal). In addition, we target *Invitational Priority 1* by accelerating math learning of students experiencing math difficulty (MD) through a multi-tiered system of support (MTSS) framework. MTSS is designed to accelerate learning by providing increasingly intensive instructional support to students who are experiencing difficulty. Within a MTSS framework, students receive classroom instruction in Tier 1. In Tier 2, interventionists provide small-group instruction to students who need additional support beyond the math taught in the general education classroom. Tier 3 is intensified and individualized for students when Tier 2 does not meet all of their math needs.

Many students will begin the 2021-22 school year significantly behind expected learning targets, and the lag in learning is projected to be even greater in math than reading (NWEA, 2020; 2021). Schools will be facing unprecedented numbers of students experiencing difficulty in math. For those schools implementing MTSS, they are actively and aggressively altering Tier 1 core instruction to support the return of students. However, many students will continue to experience difficulty in math that is not addressed by changes to Tier 1 instruction. Schools urgently need validated Tier 2 and Tier 3 interventions that are known to effectively and efficiently ameliorate lost instructional opportunities in foundational math concepts, such as fractions. Only by providing intensive and targeted intervention can schools accelerate learning for these students to support positive outcomes.

In this project, we take a What Works Clearinghouse (WWC)-approved intervention focused on fractions—*Fraction Face-Off* (*FFO*; WWC, 2020)—and contribute rigorous evidence of effectiveness with new populations and settings in Tier 2 and Tier 3 contexts. We study mechanisms to efficiently and cost-effectively scale *FFO* and build capacity within school districts to use *FFO* to improve math outcomes for students experiencing MD.

Fractions are high-priority content. Starting in Grade 3 and continuing through middle school, understanding fractions sets the foundation for algebra (National Mathematics Advisory Panel, 2008) and leads to greater success in middle school math (Bailey et al., 2012; Booth et al., 2014), high school math (Siegler et al., 2012), and college algebra (Powell et al., 2019). In middle school, fraction knowledge is widely characterized as the gatekeeper to algebra (Booth et al., 2012) and is a central contributor to future math achievement (Siegler et al, 2011; Bailey et al, 2021; Tian & Siegler, 2017). As such, intervening in fractions in upper elementary and middle school may be the most effective way of accelerating students' math learning during pandemic recovery and preparing students to be successful in high school algebra and beyond.

Students struggle with fractions. Even before the interruptions to learning caused by the pandemic, students struggled with fractions, including those who experience MD. Half of U.S. middle and high school students are not proficient with fraction concepts and procedures taught in elementary grades (National Center on Educational Statistics [NCES], 2019). Students with MD experience difficulty with fractions in 4th grade (Flores & Kaylor, 2007; Fuchs et al., 2013), 5th grade (Jordan et al., 2017), 6th grade (Hughes et al., 2018; Newton et al., 2014), 7th grade (Zhang et al., 2017), 8th grade (Mazzocco et al., 2013), into high school (Cirino et al., 2019; Dennis et al., 2016), and adulthood (Lewis, 2016), and they are more likely to experience difficulty with algebra and advanced math (Cirino et al., 2019; Dennis et al., 2016). Difficulties in STEM coursework can lead to fewer college options (Lee, 2012) and limit participation in the labor market as adults (Hawley et al., 2014; Hein et al., 2013).

Our project team, funded by an Office of Special Education Programs (OSEP) Model Demonstration Grant (2018-2022; [redacted]), worked with middle school students with MD at four sites across the U.S. during AY 2018-2019 and 2019-20. Pretest data on the *Iowa Algebra Aptitude Test* (*IAAT*) indicated students with MD scored, on average, at the 14th percentile (range = 1st to 55th). An area of greatest difficulty, as noted by participating teachers and confirmed with data, was a fundamental lack of understanding about fractions. Similar low fraction scores for students with MD have been reported in the literature (Bouck et al., 2017; Hughes et al., 2018; Losinski et al., 2019). There is a critical need to support fraction understanding in upper elementary and middle school.

Targeted intervention can improve math outcomes. A substantial body of research over the past 30 years documents elements of effective math instruction for students with MD (c.f., Dennis et al., 2016; Gersten et al., 2009; Hughes et al., 2014; Jitendra et al., 2017; Stevens et al., 2018). Consistently, explicit and systematic instruction is shown to produce positive effect sizes for a variety of outcomes including fractions, problem solving, algebra, and general math, especially when combined with heuristics, math discourse, and visual representations (often with concrete and abstract representations). Jitendra et al. (2017) noted that instructional time moderated the relationship, observing stronger effects when students received greater than 10 hours of well-designed intervention in addition to the core instruction they receive in Tier 1.

There are very few validated interventions for fractions. Successful implementation of MTSS is predicated on the availability of evidence-based interventions. However, there is a dearth of interventions available in upper elementary and middle school to expedite students' fraction knowledge and help them be ready for algebra. We conducted a synthesis of math interventions available for middle-school students with MD (Powell et al., 2020). Of the 51 studies published since 1990, only 5 focused explicitly on fractions. The majority of interventions provided support in problem solving or fluency and computation. Because competence with fractions predicts later math achievement (Bailey et al., 2012; Siegler et al., 2012), especially achievement related to algebra (Booth et al., 2014; Siegler et al., 2011), it is imperative that schools implementing MTSS have access to effective and cost-efficient interventions designed to increase student knowledge about fractions.

(2) Potential Contribution to Increase Knowledge of Effective Strategies

We address this urgent situation. We systematically replicate and extend *Fraction*

***Face-Off* for students experiencing MD**. *Fraction Face-Off* (abbreviated as *FFO;* Fuchs et al., 2013) is an intervention designed by Fuchs, Schumacher, Malone, and colleagues for students with MD in Grade 4 that meets WWC design standards without reservations with an effectiveness rating of potentially positive effects with improvement indices of +24, +31, and +33 (Fuchs et a., 2013; WWC, 2020). Please note, our research team ([redacted] and [redacted]) did not develop *FFO*, and we have never conducted research on *FFO*. We have no financial interest in *FFO*.

FFO is a small-group Tier 2 intervention designed to promote fraction understanding for students with MD. *FFO* includes 36 sessions divided into a 12-week implementation schedule, with 3 sessions per week, each lasting about 30-35 min. The intervention uses components of explicit instruction to address two types of understanding about fractions: the part-whole interpretation and the measurement interpretation of fractions. Both of these interpretations are essential for fraction competence (Jordan et al., 2013; Liu, 2018; Schumacher et al., 2018).

Each *FFO* session consists of 5 activities: (1) Training, (2) The Relay, (3) Fraction Sprint, (4) Individual Contest, and (5) Scoreboard facilitate by the interventionist.

- *Training* introduces new fraction concepts and skills and lasts 8-12 min. Tutors use guided practice and worked examples to demonstrate how to solve different fraction problems. Tutors use concrete manipulatives (e.g., fraction tiles), visual representations, and problem-solving strategies. Students focus on magnitude by comparing two fractions, ordering three fractions, placing fractions on the number line, and finding equivalent fractions.
- *The Relay* lasts 8-12 min where students take turns solving fraction problems and applying learned concepts taught during Training. The Relay also includes a cumulative review.
- *Fraction Sprint* lasts about 2 min each session. Fraction Sprint aims to build fluency on a previously taught concept or skill with three different flashcard activities.
- *Individual Contest* (5 min) and *Scoreboard* (1 min) consist of students completing word-problems and computation problems individually to demonstrate their learning of the fraction concepts. Tutors score students' work and provide corrective feedback for errors.

Evidence indicates that *FFO* is effective. Students demonstrated significant gains in comparing fractions (ES = 1.15), explaining fractions (ES = 0.76), and fraction word problems (ES = 1.18; WWC, 2020). To date, several studies at Grade 4 (Fuchs et al., 2013; Fuchs et al., 2014; Fuchs et al., 2015; Fuchs, Schumacher, et al., 2016; Fuchs, Sterba, et al., 2016; Fuchs et al., 2017; Krowka & Fuchs, 2017) have tested the efficacy of *FFO* and moderating components of *FFO* with *FFO* students scoring significantly higher on fraction outcomes when compared to students who did not participate in *FFO*. As examples, in Fuchs et al. (2013), ESs on conceptual and procedural fraction outcomes ranged from 0.29 to 2.50. In Fuchs et al. (2016), ESs on fraction understanding and fraction calculation outcomes ranged from 0.44 to 1.22. *FFO* was developed by Lynn Fuchs and colleagues at Vanderbilt University. As stated, no one on this submission team was involved with *FFO* development or implementation.

We replicate and extend research on *FFO*. First, we propose to conduct an efficacy replication at Grade 4 with diverse populations and settings. We work within authentic MTSS service delivery systems with school-based personnel. We will work with general education teachers, special education teachers, or math specialists who provide Tier 2 or Tier 3 support to students with MD (we refer to these educators as **interventionists**). Second, we build on models of effective virtual professional learning to design and test virtual *FFO* training for interventionists. Third, we investigate the effectiveness of *FFO* for improving outcomes for students with MD in subsequent grade levels. To date, all research on *FFO* has been conducted with students at Grade 4. However, there are many students in subsequent grade levels who need Tier 2 and Tier 3 intervention on fractions to achieve success with later grade math including algebra. If *FFO* leads to improved outcomes across Grade 5-8, schools will have an efficacious intervention option to use with students across several grade levels.

B. Strategy to Scale

(1) Identification of a Strategy that Addresses Barriers that have Prevented Scale

There are several limitations with existing research on *FFO*. Fuchs and colleagues conducted all initial efficacy studies of *FFO* at Grade 4 in suburban and urban schools in Nashville, TN with graduate students as interventionists. These practices are common when examining initial efficacy but may limit the generalizability of results to authentic school contexts. All training of interventionists was done in-person, which costs both time and money.

Barrier 1: One research team developed and tested *FFO* with grant funding. Fuchs and colleagues developed *FFO* with grant funding from the Institute of Education Sciences. After developing *FFO* and finishing the grant, they conducted no further implementation of *FFO*. They never attempted to scale *FFO* because of a focus on other grant projects. **Strategy 1: Replicate and extend the *FFO* research of Fuchs and colleagues.** Our research team (again, with no relationship to Fuchs and *FFO* other than collegial) plans to replicate *FFO* in Grade 4 with demographically diverse students in rural, suburban, and urban areas around Dallas, TX, Austin, TX, Columbia, MO, Kansas City, MO, and St. Louis, MO. Moreover, we explore whether *FFO* could be a mechanism for accelerating learning for students with MD in Grade 5 and middle school with similar deficits in fraction understanding, who are at greater risk for school failure. If our results demonstrate efficacy of *FFO* with students in Grade 5-8, the scalability of *FFO* improves dramatically. Schools could provide intervention to students based on need (i.e., Grade 5 and 6 students with MD) instead of by grade level. Such findings could also expand the Tier 2 and Tier 3 math interventions available for use in middle school, which is absolutely needed (Powell et al., 2020).

Barrier 2: University-based graduate students conducted all *FFO* tutoring. In Fuchs et al. (2013) and all subsequent research studies about *FFO*, the intervention was delivered by graduate students, who were hired and trained by the authors. Although this approach is commonly used in initial efficacy studies, it is untenable for scaling a school-based intervention for several reasons. First, graduate students are considerably more costly than hiring additional personnel. According to the Vanderbilt University College of Education website, annual costs for obtaining a Master's degree is about $65,000 and graduate students' stipends ranges from $14,000-30,000 per year for about 20 hours/week of support. Using the median of $22,000 and tuition costs, the hourly rate for a graduate student is about $107. However, the average hourly rate for a teacher's assistant in Tennessee is about $11/hour. Second, hiring graduate students does not build capacity to accelerate students' learning within existing school-based personnel. **Strategy 2: Train school-based interventionists to implement *FFO.*** In the proposed research, we work with existing school-based personnel who are responsible for delivering intervention within an MTSS framework to accelerate learning for students with MD. Across the four years of implementation, existing school-based interventionists will receive training in person, as originally prepared by Fuchs et al. (2013), or via virtual training using materials prepared by our research team. After each study year, business-as-usual (BAU) interventionists will receive *FFO* training. Once interventionists are trained, they should be fully capable of implementing *FFO* with multiple groups of students with MD across years, thereby increasing the capacity of existing school-based personnel for delivering an efficacious intervention for students with MD.

Barrier 3: Training costs (money and time) for *FFO* are high. In-person training presents barriers to scaling, as it may require travel from both interventionists and professional develop (PD) leaders, scheduling at times when all participants are available, and limits the number of participants a PD leader can support. Typical *FFO* training occurs in one school over 3 days with 6 hours of training each day, costing $6,000 plus travel expenses per school. With the high costs of training alone, it is not surprising that few schools pay for the *FFO* training, especially when only a few educators in a school may be providing math intervention. Also, the current *FFO* training is conducted in-person which limits the number of interventionists who can attend the training and does not account for those who have to work virtually. **Strategy 3: Provide virtual training in place of in-person professional development (PD).** We develop and test virtual training for *FFO* that interventionists can access on our project-specific website at any time. Most in-person PD is a short duration workshop that requires travel, meeting times, and cost (McConnell et al., 2013). Virtual PD "does not have to be one size fits all" and allows participating teachers to efficiently spend only as much time as they need (Fishman et al., 2013, p. 435). Similar outcomes of high fidelity have been found in teachers who received either in-person or virtual PD on a specific skill (Fisher et al., 2010). Notably, although the body of research is limited, two studies examining the impact on student outcomes of teachers who received virtual or in-person PD showed no appreciable differences in students' learning outcomes between modalities (Fishman et al., 2013; Webb et al., 2017). We create a series of brief video-based modules that explain and demonstrate the *FFO* lessons. We collect videos of interventionists using *FFO* in Year 1 to include in the modules. We record all the modules in Years 1-2 in the Lightboard recording studio at the University of Texas at Austin. We will incorporate interactive activities for interventionists to practice using the *FFO* materials. Interventionists can interact with the modules at their own pace and repeat any modules in which they need additional practice. [redacted], one of the original authors of *FFO*, has agreed to provide input on the virtual training materials (support letter in Appendix C). If virtual training is as effective as in-person training, these resources will be made freely available on the project website (hosted in perpetuity by SMU) to participating schools to support sustainability and scalability of *FFO*. We will work with the publishers to provide these training materials at no cost for schools who subsequently purchase *FFO*.

(2) Mechanisms for Broad Dissemination

With a dissemination goal of enhancing awareness and training on evidence-based fraction interventions, we will embed several strategies for increased awareness for school personnel and researchers. At the beginning of the project, we will create a project-specific website to post virtual training modules and materials for participating interventionists (similar project websites have been created at smu.edu/RME). These materials will be provided free-of-charge for participating schools. We will monitor website statistics (e.g., subscribers, views, watch time) to determine what content users are engaging with. We will use these statistics to inform updates to the site. SMU will maintain the website once funding has concluded. We create project-specific social media accounts on Twitter and Facebook.

We disseminate broadly to researchers and practitioners within special education and math education communities. We build on our successful track record of publishing in peer-reviewed research- and practitioner-oriented journals as well as presenting at related conferences. We share our findings in journals such as *Learning Disabilities Research & Practice*, *Journal of Special Education*, *Exceptional Children*, and *Journal of Mathematics Behavior*. We share findings at national conferences, such as Council for Exceptional Children, American Educational Research Association, and National Council of Teachers of Mathematics. To benefit our regional communities, we share findings at conferences such as Missouri Council of Administration of Special Education conference, the Research in Mathematics Education Conference (Dallas, Texas), and Advancing Improvement in Education Conference in Texas.

To extend the reach to practitioner communities, we conduct several webinars each year to describe the teaching of fractions to students with MD. Webinars will be provided free of charge to participants, advertised through social media and conference websites. We collaborate with existing partners at the National Center for Intensive Intervention to increase our reach (see letter of support in Appendix C). We will collect data on the number of participants in each session and will send out an evaluation after each webinar. Findings of SCALE for *FFO* will be presented in teacher-friendly terms, and we will encourage teams of educators to present with us both on the webinars and at regional or national conferences.

C. Quality of Project Design

(1) Conceptual Framework Underlying SCALE for *FFO*

This project is based on the underlying theory of change that *FFO*, when implemented with fidelity, will lead to improved outcomes for both students and interventionists as measured by standardized tests and researcher-developed measures (see Logic Model in Appendix G). We expect improvements in student outcomes will hold for Grade 4 – the grade at which *FFO* has been previously tested – and at additional grades (5-8), and will persist over time. We hypothesize that interventionists will increase their sense of self-efficacy for supporting students with MD and improve their fraction knowledge through participation in *FFO.* We hypothesize that interventionists will implement with the same degree of fidelity whether the training is delivered virtually or in person.

(2) Goals, Objectives, and Outcomes of SCALE for *FFO*

The primary purpose of this project is to replicate and expand *FFO* (Fuchs et al., 2013) with new populations and in diverse settings. Through the planned goals and objectives, we contribute evidence of effectiveness with rigorously designed studies in a wide range of communities across two states, including urban, suburban, and rural geographies as well as diverse student groups (e.g., based on socio-economic status, race and ethnicity, disability diagnosis, English learner classification). Because *FFO* is not commonly implemented by educators, we also seek to understand barriers and facilitators to implementation, including studying the impact of providing cost-effective virtual training to interventionists. We reach **our three primary aims** through four years of school-based implementation beginning in AY 2022-23 (see timeline by objective in Tables 1, 2, and 3).

AIM 1. Based on the potential evidence of effectiveness for improving fractions outcomes for students with MD, in 2022-23, we conduct an efficacy replication of *FFO* (Fuchs et al., 2013) with students with MD in Grade 4 using a regional sample of students from diverse student populations and varied geographic communities across 2 states, and implemented by school-based personnel. We recruit 20 schools with an average of 4 interventionists each (each working with 3-5 students with MD). Interventionists will be randomly assigned to condition: *FFO* or BAU. *FFO* school-based interventionists will be trained to implement *FFO* using in-person training materials provided by the developer ([redacted]; see letter of support in Appendix H), and will implement the intervention as intended with students with MD.

We evaluate changes in students' fractions knowledge and general math outcomes as a result of participating in *FFO*. Persistence of student outcomes will be monitored over the 3 remaining years of the project as students move to grades 5 (2023-24), 6 (2024-25), and 7 (2025-26). Changes in interventionists' knowledge of fractions and self-efficacy for providing math instruction to students with MD will be evaluated. Fidelity of *FFO* implementation and BAU instruction will be routinely monitored.

Table 1. Objectives, Timelines, and Roles/Responsibilities for Aim 1

Aim 1: Efficacy Replication in Gr 4	Spr 22	22-23	23-24	24-25	25-26	Fall 26	Who is Responsible?
Recruit 20 schools; assign interventionists to *FFO*, BAU	Gr 4						PDs; Coord
FFO Consultant trains staff	X						PDs; RAs
Train *FFO* interventionists; develop *FFO* fidelity checklist	Gr 4						PDs; RAs
Recruit, train data collectors	X						SMU data manager
Recruit, screen for students with MD, randomly assign		Gr 4					Coord; RAs
Implement *FFO*, collect pre-post data (*FFO*, BAU), monitor *FFO* fidelity and BAU instruction, conduct interviews		Gr 4					Interventionists; SMU data collectors; AIR
Collect follow-up data			Gr 5	Gr 6	Gr 7		RAs
Data analysis, dissemination			X	X	X	X	PDs; AIR

PD: Project Directors at SMU, MU, UT; RA: Research Assistants; Coord: Coordinator

AIM 2. To study the impact of implementing cost-effective training (virtual training), in 2023-2024, we conduct a replication of *FFO* with students with MD in Grade 4 while varying the delivery mode of training. We recruit 40 schools (each with an average of 4 interventionists per school) with diverse student populations and geographic settings in 2 states. Interventionists will be randomly assigned to 1 of 3 conditions: *FFO* with in-person training, *FFO* with virtual training, or BAU. Similar to Aim 1, we work with a regional sample of students with MD in diverse settings and communities across 2 states. We examine the relative effectiveness of virtual and in-person PD when implementing *FFO*.

We evaluate changes in students' fractions knowledge and general math outcomes during the implementation year, and over the 2 remaining years of the project as students move to grades 5 (2024-25) and 6 (2025-26). Changes in interventionists' knowledge of fractions and self-efficacy for providing math intervention to students with MD will be evaluated. Fidelity of *FFO* and BAU instruction will be routinely monitored.

Table 2. Objectives, Timelines, and Roles/Responsibilities for Aim 2

Aim 2: Replication in Gr 4 varying delivery mode of training	Spr 22	22-23	23-24	24-25	25-26	Fall 26	Who is Responsible?
Recruit 40 schools; assign interventionists to *FFO* virtual trng, *FFO* in-person, or BAU		Gr 4					PDs; Coord
Develop and refine *FFO* virtual training; create project website	X	X	X				PDs; *FFO* Consult; RAs
Train *FFO* interventionists (virtually, in-person)		Gr 4					PDs; RAs
Recruit, train data collectors		X					SMU data mgr
Recruit, screen for students with MD, randomly assign			Gr 4				Coord; RAs
Implement *FFO*, collect pre-post data (*FFO*, BAU), monitor *FFO* fidelity & BAU, conduct interviews			Gr 4				Interventionists; SMU data collectors; AIR
Collect follow-up data				Gr 5	Gr 6		RAs
Data analysis, dissemination				X	X	X	PDs; AIR

PD: Project Directors at SMU, MU, UT; RA: Research Assistants; Coord: Coordinator

AIM 3. Due to the lack of efficacious Tier 2 and Tier 3 interventions focused on fractions at upper elementary and middle school, we investigate the effectiveness of implementing *FFO* with students with MD in Grade 5 during 2024-25 and middle school (Grades 6, 7, and 8) during 2025-26. We recruit 20 schools per year, each with an average of 4 interventionists per school. Interventionists will be randomly assigned to condition: *FFO* or BAU. *FFO* training will be administered using the most efficacious training mode as determined in Aim 2.

We evaluate changes in students' fractions knowledge and general math outcomes during the implementation year. Grade 5 students will be monitored in Grade 6 during 2025-26. Changes in interventionists' knowledge of fractions and self-efficacy for providing math intervention will be evaluated. Fidelity of *FFO* and BAU instruction will be routinely monitored.

Table 3. Objectives, Timelines, and Roles/Responsibilities for Aim 3

Aim 3: Examine effectiveness for students with MD in Gr 5-8	Spr 22	22-23	23-24	24-25	25-26	Fall 26	Who is Responsible?
Recruit 20 schools; assign interventionists to *FFO* or BAU			Gr 5	Grs 6-8			PDs; Cr
Train *FFO* interventionists (use optimal mode from Aim 2)			Gr 5	Grs 6-8			PDs; GRAs
Recruit, train data collectors			X	X			SMU data manager
Recruit, screen for students with MD, randomly assign				Gr 5	Grs 6-8		Cr; GRAs
Implement *FFO*, collect pre-post data (*FFO*, BAU), monitor *FFO* fidelity and BAU instruction, conduct interviews				Gr 5	Grs 6-8		Interventionists; SMU data collectors; AIR
Collect follow-up data					Gr 6		SMU/MU/UT GRA
Data analysis, dissemination					X	X	PDs; AIR

PD: Project Directors at SMU, MU, UT; GRA: Graduate Research Assistants; Cr: Coordinator

Implementing *FFO* in Schools. To accelerate learning for students with MD, we partner with schools who have independently decided to implement MTSS (see Appendix C for letters of support). As such, these schools have already designed their master schedules to provide time for intervention support and identified the school-based personnel (e.g., interventionists) who will provide intervention. *FFO* will be implemented during the school day at the specified intervention time. No changes will be made to the publisher-specified implementation procedures: *FFO* will be administered in small groups of 3-5 students for 3 sessions per week lasting approximately 30-35 min per session. Consistent with the MTSS framework, students receiving intervention will also receive core instruction in the general education classroom.

Identifying Students with MD. In the U.S. approximately 19% of Grade 4 students and 31% of Grade 8 students fail to meet basic levels of math proficiency as determined by the National Assessment of Educational Progress (NCES, 2019). Because we are working with schools who are already implementing MTSS, we anticipate that they will have a screening system in place. However, to ensure comparability across sites, we will identify eligible students using a screening process described in the Quality of Project Evaluation section.

Selecting Interventionists. Because we work with schools who independently decided to implement MTSS, they have already identified and hired school-based personnel who will provide interventions. We will work directly with and provide training for these interventionists. Given the significant loss of learning opportunities due to the pandemic, we expect each school will have multiple sections of intervention in math to accelerate learning for students with MD. Because each school will select their interventionists based on their context and specific circumstances, we expect that the interventionist qualifications may differ. These may be general education teachers, special education teachers, paraprofessionals, math specialists, or MTSS coordinators – as noted, we call them interventionists. The publisher notes that *FFO* can be administered by paraprofessionals, teachers, or other specialists (e.g., math specialist). We will collect the interventionists' qualifications and prior experiences.

***FFO* Training.** The co-author and developer of *FFO*, Dr. [redacted], will provide initial training in a trainer-of-trainers model to Project Directors [redacted], and [redacted] and the research assistants (RA). She will also provide follow-up training support throughout the duration of the project at no cost. The Project Directors will work directly with the interventionists and research assistants will provide ongoing coaching, thereby saving the project over $600,000 in training costs (see letter of support in Appendix H).

Prior to implementation, consistent with the training recommendations of the publisher, interventionists will receive 3-days of professional development to learn how to implement *FFO*. The training includes an overview of the program goals and procedures, modeling of each activity by the trainer, and practice and feedback sessions for delivering the content. Training will occur during the school day; participating schools have agreed to provide substitute coverage for personnel to attend the training. During implementation, trained graduate students will monitor fidelity of implementation of *FFO* every two weeks using the *FFO* Fidelity Checklist (described below). Interventionists not reaching the fidelity threshold will receive supplemental coaching by the graduate student.

Monitoring Implementation Fidelity. Fidelity of implementation will be evaluated in two ways. First, trained graduate students will observe the interventionists every two weeks and complete the *FFO* Fidelity Checklist (developed in Year 1 with specific, measurable indicators that align with the key aspects of *FFO*). The purpose of this checklist is to provide immediate and ongoing support for interventionists who are not implementing with fidelity (below a threshold of 80%). Second, the external evaluator (AIR) will conduct independent fidelity of implementation observations using the same checklist.

(3) Extent to Which Project Addresses the Needs of the Target Population

We serve two groups as the target population – interventionists and students with MD.

(1) **Interventionists working with students with MD**. Researchers have demonstrated that many teachers show novice and limited understanding of fractions (Chinappan & Forrester,

2014; Lo & Luo, 2012; Tchoshanov, 2011; Utley & Reeder, 2012; Van Steenbrugge et al., 2014). We hypothesize, just as student knowledge of fractions increases through participation in *FFO*, interventionists' knowledge about fractions may increase through use of *FFO*. If we demonstrate this connection, use of a high-quality intervention, such as *FFO*, could lead to long-term positive changes in interventionists' fraction knowledge. This knowledge would likely not fade even if interventionists do not continue to use the *FFO* materials.

(2) **Students with MD**, as described, experience difficulty with fractions in 4th grade (Fuchs et al., 2013), 5th grade (Jordan et al., 2017), 6th grade (Hughes et al., 2018), 7th grade (Zhang et al., 2017), 8th grade (Mazzocco et al., 2013), into high school (Cirino et al., 2019; Dennis et al., 2016), and adulthood (Lewis, 2016). Important to this project, Jordan et al. (2013) observed that Grade 4 students with MD who struggle with fractions continue to score significantly lower on fractions tasks in Grades 5, 6, 7, and 8. Relatedly, Zhang et al. (2017) found that middle school students with MD were less accurate locating fractions on a number line and used more faulty strategies than their peers. However, evidence suggests that students with MD can improve their overall competence with fractions by developing their fraction magnitude knowledge (Tian et al., 2017).

<u>*FFO* supports students' overall proficiency with fractions</u> by focusing on two key interpretations of fractions that are essential for fraction competence (Jordan et al., 2013; Liu, 2018; Schumacher et al., 2018): part-whole and measurement interpretations.

<u>*FFO* aligns with best practice for teaching math to students with MD</u> (WWC, 2020). In *FFO*, interventionists explicitly teach fraction concepts and procedures. This explicit modeling with embedded practice opportunities is essential for students with MD (Morgan et al., 2015; Powell et al., 2020), especially the opportunities to respond that students have during modeling and practice (Haydon et al., 2012). *FFO* emphasizes the academic language of fractions to ensure students understand all discussions and can engage in discourse about fractions (Riccomini et al., 2015; Schleppegrell, 2012). *FFO* uses multiple representations, such as fraction tiles and area models, to allow students an opportunity to develop a deep understanding

of fraction concepts and procedures (Bouck & Park, 2018; Peltier et al., 2020). Furthermore, students build fluency with fraction operations and learn how to solve fraction word problems through guided and independent practice opportunities with teacher feedback.

D. Adequacy of Resources and Quality of Management Plan

(1) Capacity to Bring to Scale

Southern Methodist University (SMU) serves as the lead for this project under the direction of PI [redacted], with two implementation partners: (1) University of Missouri (MU) led by Project Director (PD) [redacted] and (2) The University of Texas at Austin (UT) led by PD [redacted]. American Institutes of Research (AIR; [redacted]) serves as the independent evaluator. Together, we are well qualified to implement the proposed rigorous research project at a regional scale. We bring expertise in research design, math education, special education, and evaluation, and have conducted rigorous grant-funded research of similar size and scope in elementary and middle school settings. Currently, [redacted], [redacted], and [redacted]r are conducting a cross-site OSEP-funded project (2018-2022; [redacted]), providing further evidence of our potential to collaboratively execute this project.

Qualified Personnel (see CVs in Appendix B)

Dr. [redacted], PI and SMU Project Director, is a Professor and the Texas Instruments Endowed Chair in Education, and the Director of the Research in Math Education unit. Since 2006, she has secured over $22M in external funding from IES, NSF, OSEP, and other sources to develop and validate assessment and instruction systems in math to support students with MD. She is responsible for overall project direction and management.

Dr. [redacted], MU Project Director, is Professor in the Dept. of Special Education. She has been awarded close to $10M in external funding as PI/co-PI from IES, OSEP, and other sources since 2003 to develop and validate assessment and instruction systems in math and writing for students academically at-risk. She will oversee and direct the MO activities.

Dr. [redacted] UT Project Director, is an Associate Professor in the Dept. of Special

Education. She has acted as PI or Co-PI on research funded by IES, OSEP, National Science Foundation, T.L.L. Temple Foundation, and NewSchools Venture Fund. Powell has authored or co-authored over 65 peer-reviewed manuscripts. She will oversee and direct the UT activities.

Dr. [redacted], AIR Project Director, is a principal researcher. He specializes in psychometrics, statistics, and research design. He was PI of a US Department of Education grant to evaluate the impact of READ 180. Swanlund has worked with a number of states, districts, and foundations to design and implement rigorous evaluations of their programs and policies.

Qualified Institutions

SMU has an established track record of innovative research supported by outstanding researchers, high-quality library resources, and a robust technology infrastructure to implement our project goals successfully. Located in Dallas, Texas, SMU is a private secular university of approximately 12,000 students from diverse economic, ethnic, and religious backgrounds. The Simmons School of Education is ranked in the top 15 in the nation for private schools by U.S. News and World Reports (2021). SMU's 10 libraries house nearly three million volumes including over 440 online databases, 38,000 electronic journals, and 300,000 electronic books.

MU College of Education has a strong commitment to research, professional training, and service. As a member of the American Association of Universities and a university classified as a "community engaged campus" and "research university/very high" by the Carnegie Foundation for the Advancement of Teaching, MU is consistently ranked within the top 15% nationwide in the preparation of teachers and leadership personnel and educational research. In research, the college's grant expenditures in FY 2020 totaled more than $21.5 million. Department of Special Education is ranked 21st on the U.S. News and World Report, 2021 rankings.

UT Austin includes 18 different colleges and schools in which almost 52,000 students are engaged in over 270 fields of study and has more than 90 organized research units. The College of Education at UT is a leading education with 5 departments including the 6th ranked Department of Special Education. The Department of Special Education, along with The Meadows Center for Preventing Educational Risk (MCPER), have won research grants exceeding $60,000,000 over the last decade. The MCPER is co-located in the College of Education on the UT Austin campus.

AIR's mission is to generate and use rigorous evidence that contributes to a better, more equitable world. We conduct rigorous, relevant research using proven methodology to improve outcomes for people at all stages of life. By emphasizing responsiveness, flexibility, product quality, and timeliness, AIR has earned a national and international reputation for efficiently and effectively meeting clients' needs and advancing research and practice.

(2) Adequacy of the Management Plan

This project is organized into three primary aims that will be conducted over a 5-year period. SMU, under the direction of PI [redacted], is responsible for overall project direction and management. MU and UT are implementation partners, and AIR serves as an independent evaluator (Project Directors at each site are identified above). University Project Directors ([redacted]) will meet weekly via Zoom videoconferencing to ensure implementation of project activities with fidelity; AIR will participate twice per month to review evaluation efforts. All Project Directors will meet in-person twice per year – once at SMU and once at a national conference. As previously noted, Project Directors [redacted], [redacted], and [redacted] have a strong history of effective collaboration to implement school-based research initiatives, and are currently conducting an OSEP-funded project (2018-2022; [redacted]. This track-record of successful partnership will decrease the likelihood of communication challenges that might impact the project's success.

Tables 1, 2, and 3 in Section C.2 identify the timeline and responsible party by objective.

PI [redacted] will facilitate implementation of the rigorous research activities designed in collaboration with AIR. She will oversee the program activities and work directly with PDs [redacted] and [redacted] facilitate implementation at their sites. Moreover, she will work directly with AIR to facilitate their ability to conduct an independent evaluation.

PDs [redacted] will work with their site coordinators to recruit participating schools. To ensure high-quality training of interventionists, PDs will receive training from the *FFO Consultant*, Dr. [redacted], and serve as the lead trainers for the interventionists at their sites. PDs are well qualified to train interventionists; they regularly provide professional development to in-service teachers, have deep understanding of math, and have extensive experience designing, implementing, and evaluating intervention research.

SMU Data Manager (Dr. [redacted]) will oversee data collection and storage for all project data. She will work directly with site coordinators at SMU, MU, and UT to ensure proper data handling procedures are followed and confidentiality is maintained, in accordance with the Institutional Review Board policies and requirements. Dr. [redacted] has over 17 years of experience conducting research that informs policy and evaluates the effectiveness of educational programs. By assigning these responsibilities to an experienced senior research scientist, the likelihood of maintaining confidentiality and data integrity is increased.

To aid in data collection and management, [redacted] will work directly with SMU's Data Center. The Data Center supports general computing and collaboration with other research partners and is designed to (a) facilitate acquisition of data, (b) develop tools and procedures for ensuring and monitoring the accuracy and confidentiality of data, (c) facilitate communication and compiling of data, (d) provide confidential storage of data, and (e) provide statistical support. SMU provides network storage to include mapped network drives accessed through the domains my.SMU and BOX.smu.edu that are available only to faculty and staff and are used for secure data storage; data are stored in a secure folder to which only key project personnel have access. These servers are routinely backed up to prevent data loss. Data will be permanently deleted using a secure data deletion software; hard-copy data will be securely destroyed.

Site coordinators will recruit interventionists, schedule training, schedule pre- and post-testing, and plan fidelity observations. Site coordinators across the three locations have extensive experience working on grant-funded research projects, have existing relationships with local schools, and are well-versed in implementing and evaluating interventions.

Graduate students with experience in math and/or special education will be recruited at each site as research assistants. We will use recruitment activities that encourage applicants from underrepresented populations. Further, the collaborating universities affirm and actively protect the rights of all individuals to equal opportunity in education and employment without regard to race, color, sex, national origin, age, religion, marital status, handicap, veteran status, sexual orientation, or any other extraneous consideration not directly and substantively related to effective performance. As applicable, the project will make efforts to employ and advance in employment qualified individuals with disabilities (see section 606 of IDEA). Research assistants will support data collection activities, coach interventionists, and monitor fidelity (*FFO*) and BAU instruction. They will receive training from the *FFO Consultant*, and will work directly with the PDs to develop skills needed to monitor fidelity and provide coaching.

Described in greater detail below, AIR will conduct an independent evaluation of the implementation, impact, scalability, and cost-effectiveness of *FFO* for students in Grades 4–8.

(3) Costs are Reasonable

The costs associated with this project are reasonable with respect to both project objectives and significance. This project provides resources to implement *FFO* in a total of 100 elementary and middle schools, thereby directly impacting about 200 teachers and 800 students with MD. After each year’s study, we provide training to interventionists working with the control group, increasing these numbers to 400 teachers and 1,600 students. As such, if effects from prior studies replicate with diverse student populations and in diverse settings, we can expect to observe 1,600 fewer students with persistent difficulties in fractions. More importantly, because fraction knowledge is described as the gatekeeper of algebra (Booth & Newton, 2012) and the purpose of this project is to contribute rigorous evidence of effectiveness, if previous findings replicate and persist, the long-term outcomes for students with MD leaving middle school with the skills needed to be successful in high school algebra cannot be overstated.

This project makes a significant investment in human capital within schools. At the end of the project, interventionists will be trained on *FFO* and can implement with subsequent cohorts of students with MD, significantly lowering the per pupil costs. Also, depending on the outcome of Aim 2 (examining virtual and in-person training), interventionists will have access to the virtual training in perpetuity through free access to the SMU-hosted website. Implementation materials (e.g., guidebook, student resources) are a one-time cost and can be re-used.

E. Quality of Project Evaluation

The American Institutes for Research (AIR) will work with research and implementation staff at SMU, MU, and UT to conduct an independent, rigorous evaluation of *Fraction Face-Off* (*FFO*). The evaluation will deliver timely and actionable feedback to the partners that is essential for ongoing monitoring and improvement of program implementation during the implementation phase of the intervention, produce evidence about the effectiveness of *FFO* that will meet the What Works Clearinghouse Evidence Standards without reservations, and provide guidance about effective strategies for implementing *FFO* at scale.

(1) Evaluation Methods to Meet the What Works Clearinghouse Evidence Standards

AIR will conduct a rigorous evaluation of the implementation, impact, scalability, and cost-effectiveness of *FFO* for students in Grades 4–8, drawing on multiple data sources including surveys, interviews, documentation, administrative data, and student and interventionist assessments. The evaluation will focus on four randomized controlled trials conducted in four different samples of schools and is designed to address the research questions listed below. Prior research has established the efficacy of *FFO* for Grade 4 students, but the efficacy of *FFO* for students in Grade 5 and middle school (Grades 6–8) is unknown. Therefore, this evaluation will first replicate the efficacy study with Grade 4 students with a diverse sample across diverse settings (RQ3), and then in a separate experiment, test the relative efficacy of virtual versus in-person training for Grade 4 interventionists (RQ9). These results will then be used to inform how interventionists are trained for the study of *FFO*'s efficacy with Grade 5 (RQ4) and middle school (RQ5) students (i.e., if one approach is shown to be more effective, or if they can be considered interchangeable as they have demonstrated similar effectiveness).

Research Questions	**Study Year**	**Data Sources**
RQ1: To what extent is *FFO* implemented with fidelity?	All RCTs (academic years 2022-23 to 2025-26)	• PD Attendance Logs/ Virtual PD Log-ins • Coaching logs • Document review •Student attendance records • Observations/ FFO Fidelity Checklist
RQ2: What are facilitators of and barriers to successful implementation?	All RCTs (academic years 2022-23 to 25-26)	• Interventionist interviews
RQ3: What is the impact of *FFO* on Grade 4 students' fractions and general math knowledge?	RCT 1: 2022-23 academic year	• Student assessments
RQ4: What is the impact of *FFO* on Grade 5 students' fractions and general math knowledge?	RCT 3: 2024-25 academic year	• Student assessments
RQ5: What is the impact of *FFO* on middle school students' fractions and general math knowledge?	RCT 4: 2025-26 academic year	• Student assessments
RQ6: Does implementing *FFO* increase interventionists' fraction knowledge?	All RCTs (academic years 2022-23 to 25-26)	• Interventionist assessments

RQ7: Does interventionists' fraction knowledge mediate increases to students' fractions and general math knowledge?	All RCTs (academic years 2022-23 to 25-26)	• Student assessments • Interventionist assessments
RQ8: Does interventionists' self-efficacy mediate increases to students' fractions and general math knowledge?	All RCTs (academic years 2022-23 to 25-26)	• Student assessments • Interventionist survey
RQ9: What is the relative effectiveness of virtual PD and training versus in-person PD and training?	RCT 2: 2023-24 academic year	• Interventionist interviews and assessments • Student assessment
RQ10: What is the cost effectiveness of the *FFO* instructional intervention?	All RCTs (academic years 2021-22 to 24-25)	• Student assessments • Interventionist assessment • Cost information

Over the course of the evaluation, AIR will conduct <u>4 blocked cluster-randomized experimental studies</u> with teacher-level (interventionist) random assignment within schools to test the effectiveness of *FFO*. The combined sample size (see the power analysis below) across all studies will include 100 schools, 400 interventionists, and 1,600 students with MD. The first experiment (2022–23 academic year, 20 schools, 80 interventionists, 320 students) will focus on the impact of *FFO* for Grade 4 students with MD in various communities in two states, including urban, suburban, and rural geographies as well as diverse student groups (e.g., based on socio-economic status, race and ethnicity, disability diagnosis, English learner classification); the second experiment (2023–24, 40 schools, 160 interventionists, 640 students) will examine the relative effectiveness of virtual and in-person PD when implementing *FFO*, again with a diverse sample of Grade 4 interventionists and students with MD; the third experiment (2024–25, 20 schools, 80 interventionists, 320 students) will examine the impact of *FFO* for Grade 5 students with MD; and the fourth experiment (2025–26, 20 schools, 80 interventionists, 320 students) will examine the impact of *FFO* for middle school students (Grades 6–8) with MD. The main confirmatory analysis for each of these experiments will focus on treatment/control contrasts based on pre- and post-assessments administered within the academic year in which students receive the *FFO* intervention. However, treatment and control students will continue to be follow-up tested in subsequent school years (through the end of the grant) to explore the long-term effects associated with *FFO*.

Student eligibility to receive the intervention will be determined via a screening process prior to randomization. Students with an identified learning disability and IEP goals in math and who score below the 35th percentile (based on normative data) on the *Wide Range Achievement Test*-5 (WRAT-5) Math Computation are the target population. After obtaining caregiver consent, we screen students using the WRAT-5 and determine the list of eligible students.

We then randomly assign interventionists, within school, to either the *FFO* condition or a business-as-usual (BAU) condition. For the experiment focused on the relative effectiveness of virtual versus in-person training, interventionists will be randomly assigned within school to *FFO* with virtual training, *FFO* with in-person training, or BAU. The experiments will measure and track outcomes for all students who met eligibility criteria in treatment and control groups prior to random assignment. This process will allow us to track any joiners to the intervention and control groups, along with any treatment group crossover. The primary impact analyses will not include joiners, per WWC Evidence Standards. Furthermore, we will analyze and document the baseline equivalency of our experimental groups with respect to key pre-intervention measures and covariates. Any imbalance in treatment groups will be dealt with per WWC guidelines (e.g., statistical adjustment). Due to the within-school random assignment of interventionists, monitoring for spillover bias will be imperative. To mitigate the potential for contamination of the BAU group, materials and training for *FFO* will be given only to treatment interventionists (control interventionists will receive the materials and training after the intervention year). In addition, we will also use classroom observations (with the *FFO* Fidelity Checklist), teacher surveys, and interventionists interviews to determine whether aspects of the treatment existed in the control classrooms.

Across the 4 experiments, we plan for a total sample of 100 schools, with 3 of the experiments requiring 20 schools, and the fourth experiment, with 2 treatment conditions requiring 40 schools. We plan for independent samples of 20 schools for the Grade 4, Grade 5, and middle school experiments, with an average of 4 interventionists per school (split between the treatment and BaU conditions). For the experiment examining the relative effectiveness of virtual versus in-person training, we will plan for a sample of 40 schools given that we will have 2 experimental contrasts (two different treatment groups, with a single control group).

These planned sample sizes were based on power analyses conducted using the PowerUp! software (Dong et al., 2015). A previous study of *FFO* for Grade 4 students documented standardized effect sizes ranging from 0.29 to 2.5 for various student outcomes (Fuchs et al., 2013). Given the wide range of observed effects, our power analysis assumed a conservative effect size target range of 0.25 to 0.29 standard deviations. This number seems appropriate given the intensity of the intervention (36 sessions of at least 30 min) and the proximal nature of most of the student assessments. Furthermore, we assumed an average of four students per interventionist, and four interventionists per school, and covariates (student pretests) that explain 50% of the variance at the student and interventionist levels. Finally, we assumed a range of 10 to 16% of the variance in outcomes attributable to classroom-to-classroom differences within schools (Schochet, 2005). These power analyses suggested the number of schools included in each experiment to have sufficient power (.80) for a two-tailed test (with alpha of .05) ranged from 16 (assuming an ICC of .10 and effect size of .29) to 24 (assuming an ICC of .16 and effect size of .25). During recruitment for each experiment, we conduct additional power analyses to ensure that the nature of the recruited sample (distribution of interventionists across schools) yields sufficient statistical power.

For each of the 4 experiments, students will be given pre- and post-assessments, with follow-up in subsequent academic years (post-test only) 1 to 3 years later. See Tables 1-3 for timeline. The confirmatory analyses of impact will be based on the pre- and post-assessments administered during the year when receive *FFO*. Follow-up differences between treatment and control students in subsequent years is considered exploratory.

The analysis of the impact of *FFO* on student outcomes will use the blocked cluster-randomized design described previously (interventionists randomly assigned within school). The main impact model for student outcomes will be a mixed-effects regression model, with the following general form:

$$Y_{ijk} = \beta_0 + \beta_1 FFO_{jk} + \alpha \boldsymbol{X}_{ijk} + \delta \boldsymbol{W}_{jk} + \pi \boldsymbol{S}_k + r_{jk} + \varepsilon_{ijk}$$

where Y_{ijk} represents the math and fraction outcomes of student *i* nested in interventionist *j* in school *k*. The treatment effect is measured by the coefficient β_1 for the treatment indicator ($FFOj_k$). The model will also include student and interventionist covariates represented by the covariate vectors $\mathbf{X}_{ijk}$ and $\mathbf{W}_{jk}$, respectively. Student covariates will include demographic characteristics and the pretest measures of the outcome being predicted by the model. Student covariates for eligible treatment and control students will be aggregated to serve as covariates at the interventionist level to further increase the precision of the treatment effect. In addition, to account for the blocked randomization of interventionists within schools, a vector of school fixed effects ($\mathbf{S}_k$) is included in the model. Covariates will be selected a priori for inclusion in the model to avoid researcher-induced bias due to covariate selection during impact modeling. The model includes a random effect for interventionists (r_{jk}) along with the student error term (ε_{ijk}). We will fit two parallel models for the experiment looking at the relative efficacy of virtual versus in-person training, and will compare the effect estimates, while adjusting confidence intervals for non-independence due to the shared control group.

Interventionists' fraction knowledge will be modeled in a similar fashion, with outcomes predicted as a function of the treatment indicator along with the interventionist pretest and school-fixed effects to account for within-school randomization. We will estimate mediator models in order to examine whether changes in interventionists' self-efficacy and math knowledge mediate the impact of *FFO* on student outcomes. Key to conducting a mediator analysis is a relationship between the treatment variable and the mediator. First, we will fit regression models that consider the impact of the treatment on these teacher-level outcomes. If a positive relationship between treatment and self-efficacy or interventionist knowledge is found, we can proceed with a mediator analysis. In the mediator model, the specific indirect effect of the treatment on the outcome via the mediator is defined as the product of the two unstandardized paths linking the treatment to the outcome via that mediator (Schochet, 2009). The effect of each mediator may be estimated using the following multilevel models:

Student-level outcomes predicted by the treatment:

$$Y_{ijk} = \beta_0 + \beta_1 FFO_{jk} + \alpha \boldsymbol{X}_{ijk} + \delta \boldsymbol{W}_{jk} + \pi \boldsymbol{S}_k + r_{jk} + \varepsilon_{ijk}$$

where Y_{ijkl} represents the academic outcome of student *i* nested in interventionist *j* in school *k*. The treatment effect is measured by the coefficient β_1 for the treatment indicator ($FFOj_k$). The model will also include student and interventionist covariates represented by the covariate vectors $\mathbf{X}_{ijk}$ and $\mathbf{W}_{jk}$, respectively.

Student-level outcomes predicted both by the treatment and the mediators:

$$Y_{ijk} = \beta_0 + c' FFO_{jk} + bM_{jk} + \alpha \boldsymbol{X}_{ijk} + \delta \boldsymbol{W}_{jk} + \pi \boldsymbol{S}_k + r_{jk} + \varepsilon_{ijk}$$

Mediator predicted by treatment:

$$M_{jk} = \beta_0 + aFFO_{jk} + \delta \boldsymbol{W}_{jk} + \pi \boldsymbol{S}_k + r_{jk}$$

The mediator effect is then calculated by multiplying the estimates a and b (after having controlled for effect c').

(2) Guidance about Effective Strategies Suitable for Replication in Other Settings

The evaluation is designed to provide both formative and summative information on effective strategies for *FFO* implementation. First, interventionist interviews and surveys will be used to determine facilitators and barriers to successful implementation as well as interventionists' views on how and why *FFO* may be effective with different groups of students with different needs. Second, we are explicitly testing two different models for PD delivery (virtual and in-person) to determine their relative efficacy. Third, the block randomized design (random assignment of interventionists within schools) will allow the evaluation team to explore the heterogeneity of the *FFO* treatment effects among schools, and link those treatment effects to variations in implementation (including the relative fidelity of implementation for subcomponents of the program delivery model), interventionist beliefs and attitudes, characteristics of the student population in each school, interventionist experience and interventionist knowledge, and the PD delivery method.

(3) Clear Articulation of Components, Mediators, and Outcomes

The *FFO* intervention includes PD for interventionists, coaching, materials for interventionists and students, and expectations around both the length and number of sessions, which are to be conducted with students. During the first year of the grant, prior to randomization for the first experiment focusing on Grade 4 students and interventionists, we will develop an implementation fidelity rubric (separate from the *FFO* Fidelity Checklist for use in classroom observations), with specific, measurable indicators for all key aspects of implementation. Clear thresholds will be set for each indicator. The rubric will measure the following aspects of implementation (with potential data sources listed): (a) **Materials:** Document review to determine the extent to which all interventionists have received instructional materials for themselves and their students (planned threshold, 100%); (b) **Training:** training attendance logs, virtual training log-ins and completion (planned threshold, 80% of training occurs); (c) **Instruction:** Interventionist documentation of number of instructional sessions and length, observation scores from the *FFO* Fidelity Checklist, student attendance/rosters (planned threshold, 80% of 36 planned instructional sessions occur).

Assessments for Students with MD and Interventionists

The 4 proposed experiments are designed to test the impact of *FFO* on student and interventionist outcomes. Student outcomes include a series of math assessments focusing on fractions and general math knowledge. Interventionist outcomes include fractions knowledge and self-efficacy for math instruction, and will be examined as mediators of student math knowledge (see the mediator analysis described above). Past estimates of reliability are reported; we will conduct additional psychometric validation of each measure and document their reliability in our study sample and ensure that reliability is sufficient to meet WWC evidence standards.

Student Measures

Fraction Understanding. Researcher-developed (Jordan et al, 2017). Administered pre-post. Measures students' fraction concepts, fraction number line estimation, and fraction arithmetic skills. Provides accurate information (AUCs > .81; Rodrigues et al., 2019).

Vanderbilt Fraction Battery. Researcher-developed (Schumacher et al., 2013). Administered pre-post. Measures faction comparison and magnitude understanding of fractions. Reliability of .90 (Fuchs et al., 2013).

Fraction Number Line. Researcher-developed (Siegler et al., 2011). Administered pre-post. Assesses the measurement interpretation of fractions by requiring students to place fractions on a number line. Reliability of .79.

Wide Range Achievement Test-5 Math Computation (WRAT-5). Standardized assessment (Wilkinson & Robertson, 2017). Administered pre-post. Measures math ability including counting, number identification, and calculation. Reliability of .88.

Stanford Achievement Test-10. Standardized assessment (Pearson, 2007). Administered pre-post. Comprehensive battery of math achievement. Reliability of .80-.87.

Iowa Algebra Aptitude Test (IAAT). Standardized assessment (Schoen & Ansley, 2007). Administered post-only. Measures algebra-related math concepts Reliability of .90.

Interventionist Measures

Fraction Number Line. Researcher-developed (adult version; Schneider & Siegler, 2010). Administered pre-post as outcome measures, also used as a mediator of student math knowledge. Measures fraction knowledge, magnitude comparison, and locating on a number line.

Mathematics Teaching Efficacy Belief Instrument. Researcher-developed (Enochs, Smith, & Huinker, 2000; MTEBI). Administered pre-post as outcome measures, also used as a mediator of student math knowledge. Measures self-efficacy related to instruction in general as well as focused on math and fractions instruction. Reliability of .75-.88.

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